Preliminary Results of Geometry Scaling and Magnetic Topology Study for Low Power Hall Effect Thrusters

Erdal Bozkurt
Middle East Technical University, Ankara, 06800, Turkey

Murat Celik
Bogazici University, Bebek, Istanbul, 34342, Turkey

Abstract

Satellites moving in orbit generally use chemical thrusters that use the principle of conservation of momentum to provide the desired thrust. Over the last several decades, there has been an increased interest in developing thrusters that use the on-board electrical energy to provide in space propulsive needs of such satellites. Among these propulsion concepts, Hall Effect thrusters are the most studied and widely employed electric thrusters. Hall Effect thrusters convert the electric power into thrust by the ionization of the propellant gas and the acceleration of the ionized gas under the influence of electric and magnetic fields. Even though, the Hall Effect thrusters have a simple geometry and are easy to manufacture, the physical working principle of these thrusters are not well understood. There are also many different Hall thrusters with different sizes and power levels. By using the proper scaling laws it is possible to predict thruster performance parameters. In this study, appropriate scaling laws are determined and a 30mm diameter SPT type Hall Effect thruster is designed. In addition, by providing the appropriate magnetic field topology it is possible to obtain higher efficiency. In this study the scaling method and the magnetic topology studies are presented.

TRK-30 Hall Effect Thruster

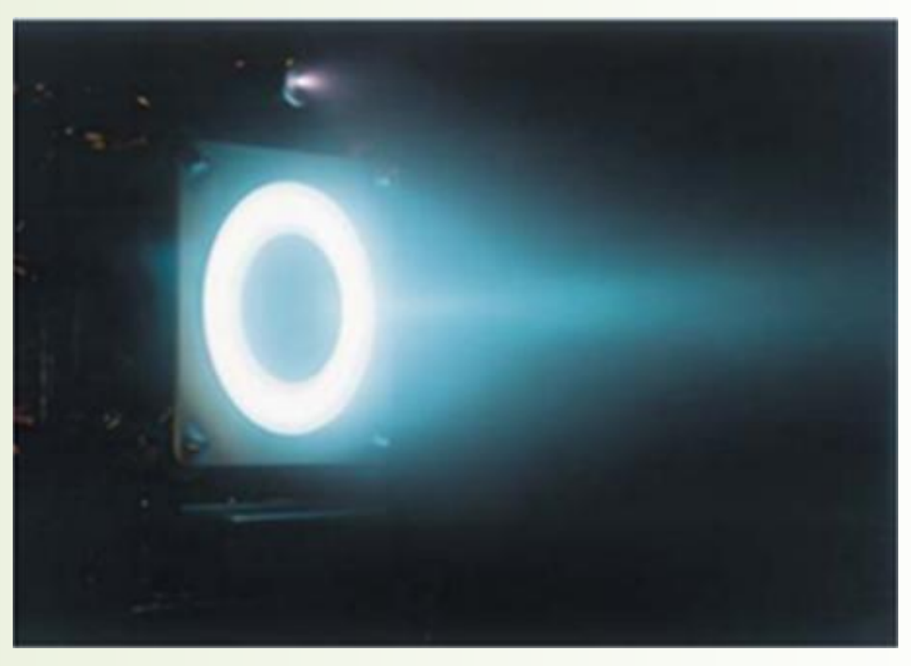


Figure: Pictıure of a Hall Effect thruster during its operation inside a vacuum chamber.

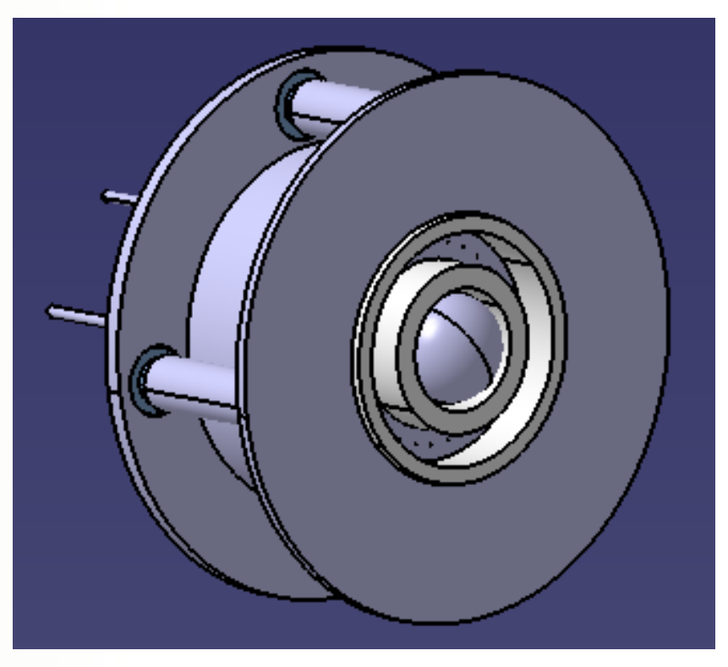


Figure: Solid CAD Model of the proposed prototype Hall Effect thruster TRK-30, in the isometric view.

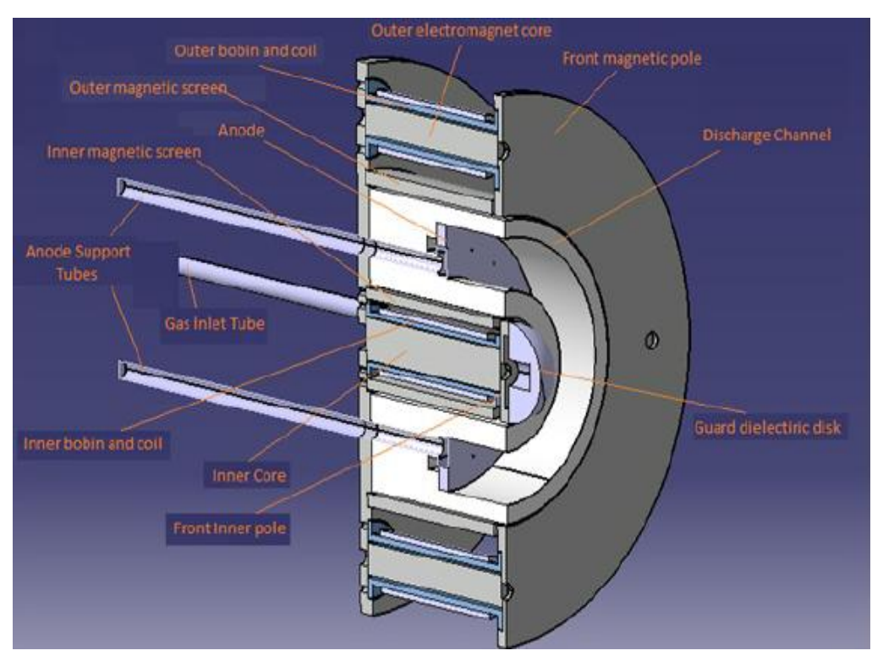
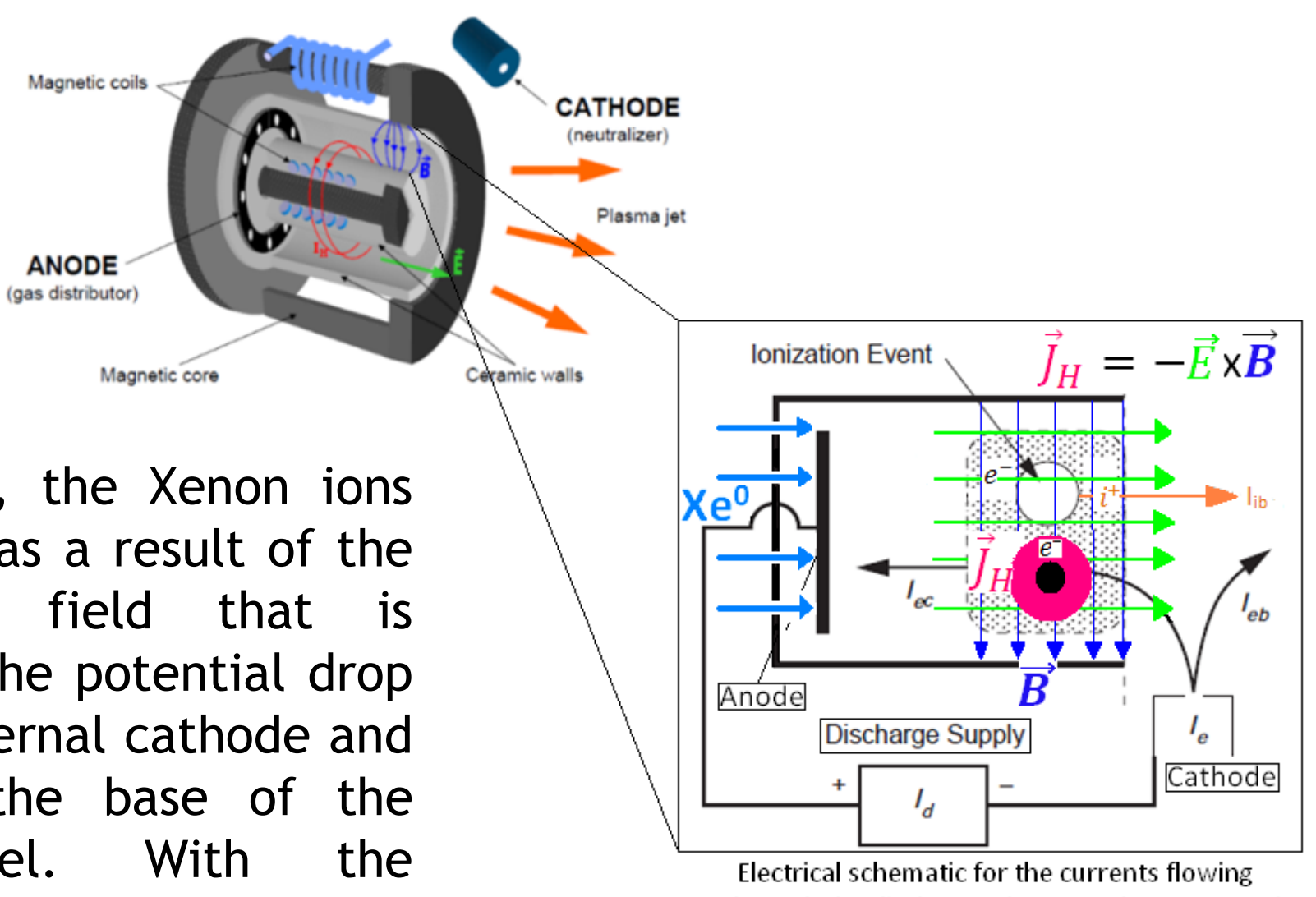


Figure: Main parts of the proposed prototype Hall Effect thruster, in cross sectional view.

Cross magnetic and electric field cause the Hall current of the electrons that are coming from the cathode. At the exit plane of the thruster electric potential drops due to trapped electrons in the Hall current. At the channel entry there is an anode and in here the neutral Xenon propellant gas atoms are delivered. Near the channel exit the released Xenon atoms are ionized by means of the electrons in the Hall current.



Electrical schematic for the currents flowing through the discharge plasma and power supply.

After ionization, the Xenon ions are accelerated as a result of the acting electric field that is created due to the potential drop between the external cathode and the anode at the base of the annular channel. With the principal of the momentum conservertion these accelarated ions providing thrust to the spacecraft.

Results of magnetic curcuit and magnetic field topography

In order to get good performance values from the proposed prototype HET, the magnetic field topology should be similar to that of the 5 kW P5 Hall Effect Thruster. P5 magnetic field topology is a mature one which is studied by the scientist of the Soviet Union working on Hall Effect Thrusters and other scientists elsewhere. It guarantees higher efficiency, high ionization level, high thrust, high specifical impulse, better optimized plume divergence angle and higher lifetime thanks to lower divergence angle. To obtain such good performance parameters, the first goal was to obtain similar magnetic flux density B distribution like that of P5. To obtain a similar magnetic field topology, for the proposed prototype thruster TRK-30 HET magnetic circuit was designed like as that of the P5 magnetic curcuit. Therefore two apperantly similar magnetic flux density distribution are seen for these two thrusters. The FEM simulations are done using COMSOL and the obtained analysis results are shown at the left upper corner below.

$$\vec{J}_e = \frac{NI_{cir}}{A}$$

$$\vec{\mathrm{B}} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\times(\mu_o^{-1}\mu_r^{-1}\vec{\mathrm{B}}) = \vec{J}_e$$

$$\vec{\nabla}\times(\mu_o^{-1}\mu_r^{-1}\vec{\nabla}\times\vec{A}) = \vec{J}_e$$

COMSOL solves the last equation with the entry of the current density and it finds the magnetic vector potential **A**. After solving magnetic vector potential, it can easily calculate the magnetic flux density **B** upon the solution domain.

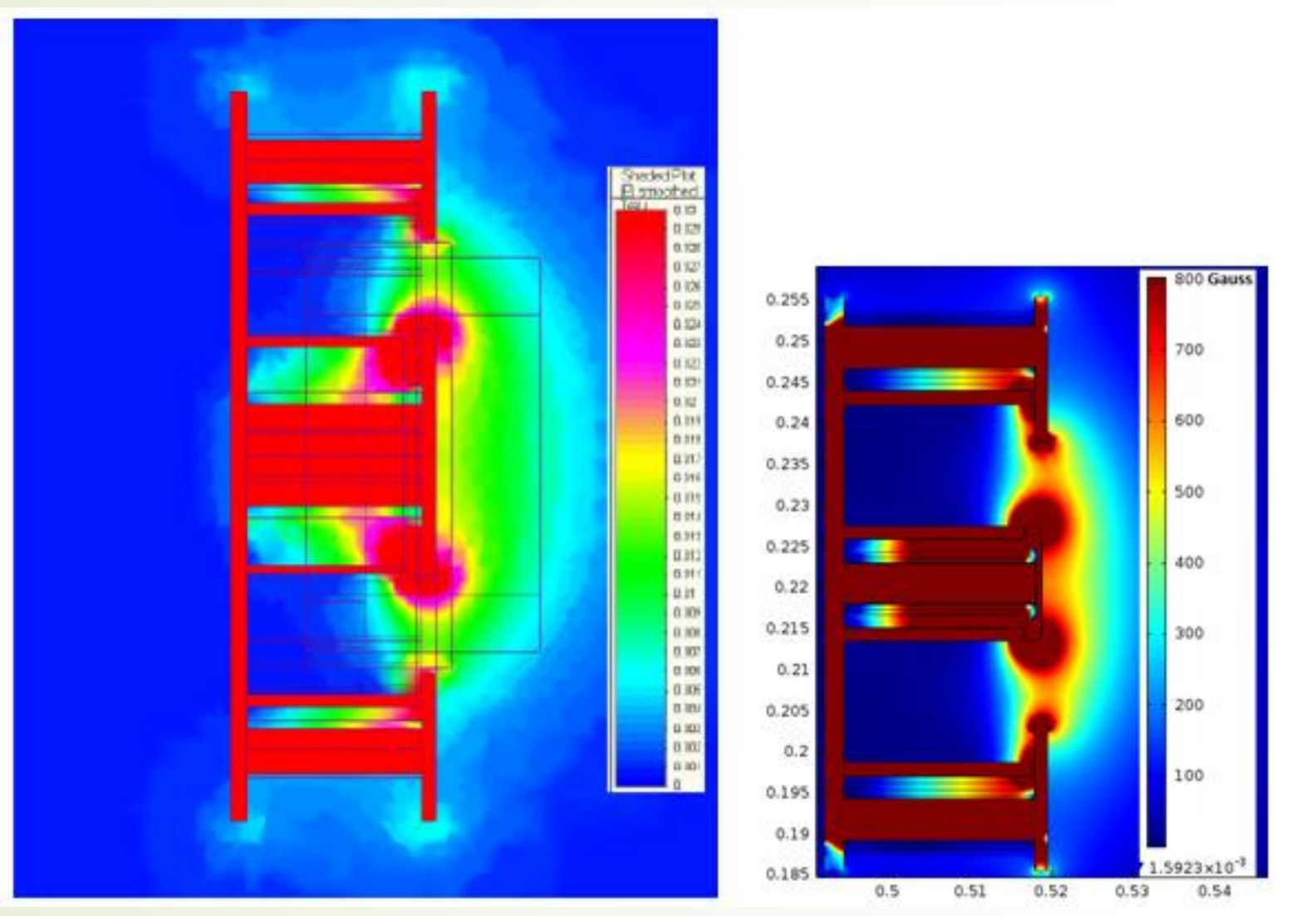


Figure: Magnetic flux density distribution at the domain, left P5(32 cm max outer diameter), right TRK-30 (7 cm max outer diameter)

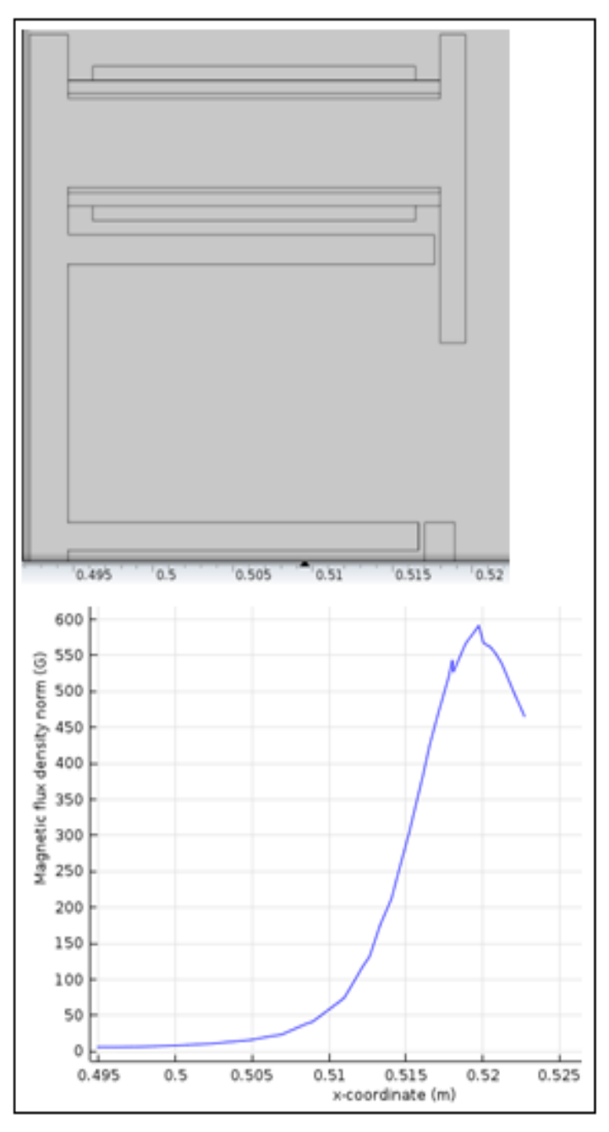


Figure: Magnetic flux density distribution in 1D

In order to obtain an efficient ionization region and therefore enough Hall current and accelaration of ions, the maximum magnitude of the magnetic field should be located approximately at the discarge channel exit plane of the HET. In the left Figure, with the proper design of the magnetic curcuit, the maximum magnitude of 600 Gauss magnetic field can be reached at the exit plane of the discharge channel and the calculations are done in the middle plane of the channel.

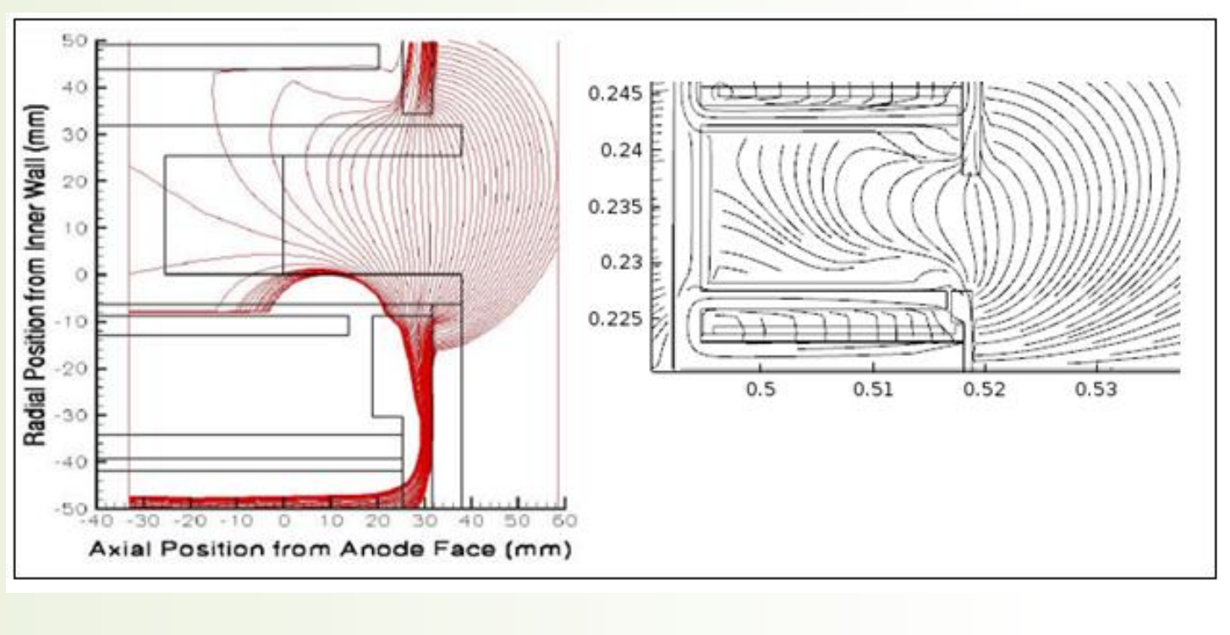


Figure: Magnetic field streamlines for the P5 left and for the right TRK-30

In order to sustain high thruster efficiency, lifetime and low ion divergence angle, the topology or distribution of the magnetic field lines are most important. In the left Figure, the aim is to reach the red color distribution of the P5 5 kW Hall Effect thruster [2]. That distribution is very mature after years of the experiences and optimizations of that field of the space propulsion. The generated distribution that is drawn in black color is very close to that of P5 but small deviations are seen again, the reason of it may be caused due to the use of different FEM solvers.

Fundamental Physical Relations for Hall Effect Thruster Scaling and Its Performance Prediction Results

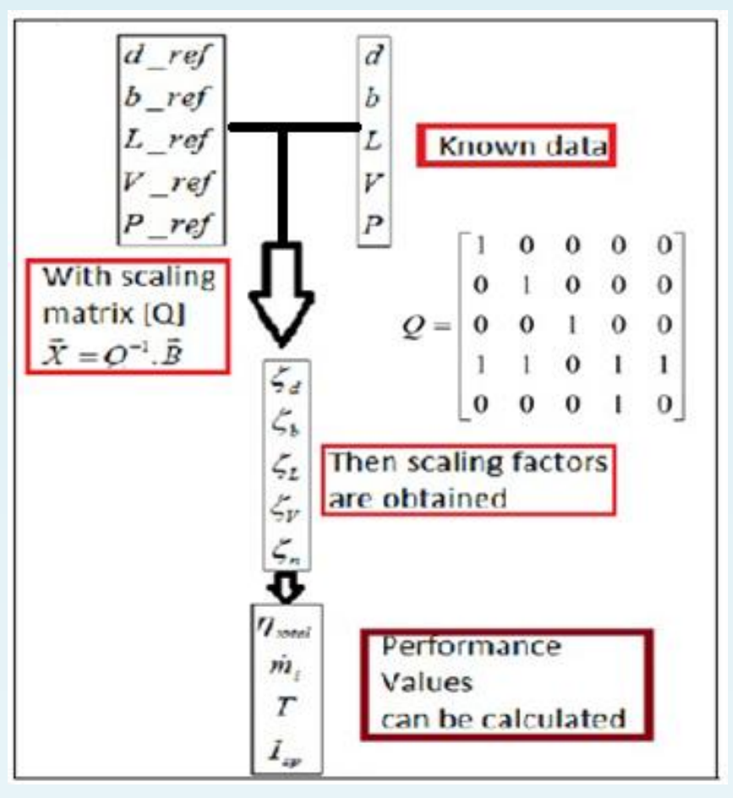


Figure 1: The scaling algorithm

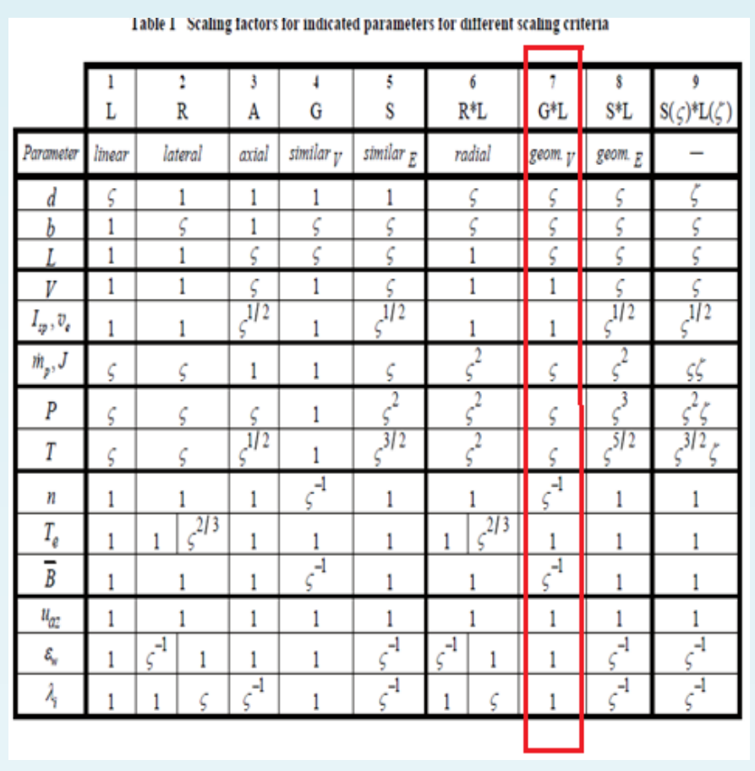


Table 1 Scaling factors for indicated parameters for different scaling criteria

	1 L	2 R	3 A	4 G	5 S	6 R*L	7 G*L	8 S*L	9 S(ς)*L(ζ)
Parameter	linear	lateral	axial	similar $_V$	similar $_E$	radial	geom. $_V$	geom. $_E$	—
d	ς	1	1	1	1	ς	ς	ς	ζ
b	1	ς	1	ς	ς	ς	ς	ς	ς
L	1	1	ς	ς	ς	1	ς	ς	ς
V	1	1	ς	1	ς	1	1	ς	ς
I_{sp}, v_e	1	1	$\varsigma^{1/2}$	1	$\varsigma^{1/2}$	1	1	$\varsigma^{1/2}$	$\varsigma^{1/2}$
$\dot{m}_p, J$	ς	ς	1	1	ς	ς^2	ς	ς^2	$\varsigma\zeta$
P	ς	ς	ς	1	ς^2	ς^2	ς	ς^3	$\varsigma^2\zeta$
T	ς	ς	$\varsigma^{1/2}$	1	$\varsigma^{3/2}$	ς^2	ς	$\varsigma^{5/2}$	$\varsigma^{3/2}\zeta$
n	1	1	1	ς^{-1}	1	1	ς^{-1}	1	1
T_e	1	1 $\varsigma^{2/3}$	1	1	1	1 $\varsigma^{2/3}$	1	1	1
$\bar{B}$	1	1	1	ς^{-1}	1	1	ς^{-1}	1	1
u_{az}	1	1	1	1	1	1	1	1	1
ε_w	1	ς^{-1} 1	1	1	ς^{-1}	ς^{-1} 1	1	ς^{-1}	ς^{-1}
λ_i	1	1 ς	ς^{-1}	1	ς^{-1}	1 ς	1	ς^{-1}	ς^{-1}

Table 1: The scaling matrix [1].

$$\begin{bmatrix} ln(\frac{d}{d_{ref}}) \\ ln(\frac{b}{b_{ref}}) \\ ln(\frac{L}{L_{ref}}) \\ ln(\frac{P}{P_{ref}}) \\ ln(\frac{V}{V_{ref}}) \end{bmatrix} = \begin{bmatrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0\\ 1&1&0&1&1\\ 0&0&0&1&0 \end{bmatrix} \begin{bmatrix} ln\zeta_d \\ ln\zeta_b \\ ln\zeta_L \\ ln\zeta_V \\ ln\zeta_n \end{bmatrix}$$

$$\vec{B} = \begin{bmatrix} ln(\frac{d}{d_{ref}}) \\ ln(\frac{b}{b_{ref}}) \\ ln(\frac{L}{L_{ref}}) \\ ln(\frac{P}{P_{ref}}) \\ ln(\frac{V}{V_{ref}}) \end{bmatrix}, \quad Q = \begin{bmatrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0\\ 1&1&0&1&1\\ 0&0&0&1&0 \end{bmatrix}, \quad \vec{X} = \begin{bmatrix} ln\zeta_d \\ ln\zeta_b \\ ln\zeta_L \\ ln\zeta_V \\ ln\zeta_n \end{bmatrix} \quad \Rightarrow \vec{X} = Q^{-1}.\vec{B}$$

In order to determine the diameter of the discharge chamber, first of all the specific impulse must be known for the mission of the satellite. In the light of the experimental works for the Hall Effect thrusters, it is known that a given specific impulse corresponds to a minimum power level that the thruster operates and the dependence between these two quantities is shown in the Figure 2. As seen from the Figure 1, 1660 sec. specific impulse corresponds to approximately 1.5 kW electric power of the SPT-100. At the same time, in order to sustain this power level, from experimental works it is known that the discharge outer diameter "d_{ch}"should be 100 mm as seen in Figure 2 like as SPT-100. On the other hand, for the proposed prototype 200 Watt power level thruster, the discharge outer diameter correspond to 30 mm experimentally.

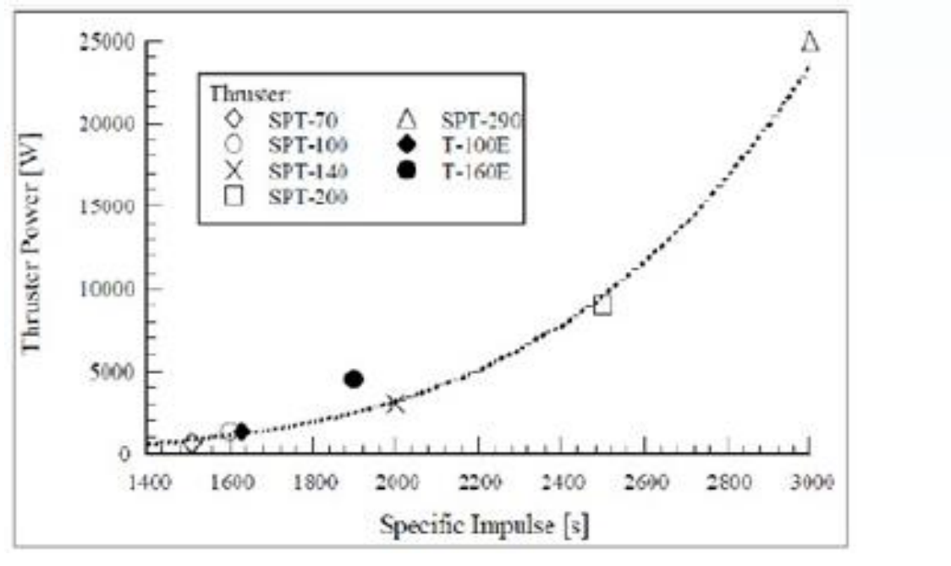


Figure 2: Specific Impulse vs Power table

According to given geometric scaling laws [1], in Table 1 the 7th colored red box, all of the scaling operation is summarized.

It is seen that, first of all, the scaling factor sould be determined. In order to calculate it, the ratio between the mean diameter of the reference thruster and proposed prototype thuster is determined. The strongest reason of why this we choose these mean diameters to calculate the ratio is that the "d" mean diameter value for HET is identical for thruster operating parameters like as power, mass flow rate, and performance parameters such as thrust and specifical impulse. On the other hand for the proposed prototype thruster TRK-30, we decided operating power point is 258 Watt. The main reason of that desicion is our vacuum chambers are small and our vacuum pumps level is low when we compare for 1.5 kW class HET that has 5 mg/sec mass flow rate. Our vacuum pumps are proper only for vacuum in the level 1.0 mg/sec mass flow rate and therefore 24 mm of mean diameter of the SPT-30 class HET is good candidate for our test facility.

$$\frac{d_{TRK-30}}{d_{ref}} = \varsigma, \quad d_{TRK-30} = 24mm,$$
$$d_{ref} = d_{SPT-100} = 85mm,$$
$$\varsigma = 0.2824$$

After calculating the scaling factor, we can calculate the other main two thruster dimensions "b" and "L".

$$\frac{b_{TRK-30}}{b_{ref}} = \varsigma,$$
$$b_{TRK-30} = \varsigma * b_{SPT-100}$$
$$b_{TRK-30} = 4,2360\,\mathrm{mm}$$

$$\frac{L_{TRK-30}}{L_{ref}} = \varsigma,$$
$$L_{TRK-30} = \varsigma * \mathrm{L}_{SPT-100}$$
$$L_{TRK-30} = 6,213\,\mathrm{mm}$$

For the proposed HET, the operating power level and voltage value are choosen from SPT-30 operating parameters. The main reason of that decision is if we choose SPT 30 operating parameters for performance prediction whether performance of the proposed prototype thruster can catch thoose of the SPT-30 or not.

$$P_{TRK-30} = P_{SPT-30} = 258\ Watt$$
$$V_{TRK-30} = V_{SPT-30} = 250 Volts$$

	SPT-100 reference HET dimensions	TRK-30 calculated dimensions and other operating parameters
d_{ch}(mm)	100	30
d(mm)	85	**24**
b(mm)	15	**4.24**
L(mm)	22	**6.213**
V(Volts)	300	**250**
P(Watt)	1350	**258**

Table 2: SPT-100 and TRK-30 comparison

Parameters	Channel mean diameter	Channel height	Channel length	Applied voltage	Gas inlet density
d/d_{ref}	ζ_d	1	1	1	1
b/b_{ref}	1	ζ_b	1	1	1
L/L_{ref}	1	1	ζ_L	1	1
V/V_{ref}	1	1	1	ζ_V	1
n/n_{ref}	1	1	1	1	ζ_n
J_D/J_{D_ref}	ζ_d	ζ_b	1	1	ζ_n
P/P_{ref}	ζ_d	ζ_b	1	ζ_V	ζ_n
B_{max}/B_{max_ref}	1	1	$(\zeta_L)^{-1}$	ζ_V	1
$\lambda_i/\lambda_{i\_ref}$	1	1	$(\zeta_L)^{-1}$	1	$(\zeta_n)^{-1}$
$\lambda_{diff}/\lambda_{diff\_ref}$	1	1	$(\zeta_L)^{-1/2}$	$(\zeta_V)^{-1/2}$	$(\zeta_n)^{-1/2}$
$\varepsilon_w/\varepsilon_{w\_ref}$	1	$(\zeta_b)^{-1}$	ζ_L	$(\zeta_V)^{-1}$	1
$\varepsilon_a/\varepsilon_{a\_ref}$	1	1	1	$(\zeta_V)^{-1}$	1
$\varepsilon_i/\varepsilon_{i\_ref}$	1	1	1	$(\zeta_V)^{-1}$	1

Table 3: Scaling matrix derived from all scaling factors.

According to scaling matrix we can write the following five equations for the unknowns $(\zeta_d, \zeta_b, \zeta_L, \zeta_V, and\ \zeta_n)$.

$$\frac{d}{d_{ref}} = \zeta_d$$
$$\frac{b}{b_{ref}} = \zeta_b$$
$$\frac{L}{L_{ref}} = \zeta_L$$
$$\frac{P}{P_{ref}} = \zeta_d.\zeta_b.\zeta_V.\zeta_n$$
$$\frac{V}{V_{ref}} = \zeta_V$$

Taking the logarithm

$$ln(\frac{d}{d_{ref}}) = ln(\zeta_d)$$
$$\ln(\frac{b}{b_{ref}}) = ln(\zeta_b)$$
$$\ln(\frac{L}{L_{ref}}) = ln(\zeta_L)$$
$$\ln(\frac{P}{P_{ref}}) = ln(\zeta_d) + ln(\zeta_b) + ln(\zeta_V) + ln(\zeta_n)$$
$$\ln(\frac{V}{V_{ref}}) = ln(\zeta_V)$$

Passing to the logarithms, it is approached that a linear svstem of equations are obtained and their solution is very easy. If we write the system in the standard form $\vec{B} = Q.\vec{X}$, indicating with Q the matrix of coefficients, we have:

$$\frac{d}{d_{ref}} = 0,2824 \qquad \frac{L}{L_{ref}} = 0,2824 \qquad \frac{V}{V_{ref}} = 0,8333$$
$$\frac{b}{b_{ref}} = 0,2824 \qquad \frac{P}{P_{ref}} = 0,1911$$

$$\vec{B} = \begin{bmatrix} \ln(0,2824) \\ \ln(0,2824) \\ \ln(0,2824) \\ \ln(0,1911) \\ \ln(0,8333) \end{bmatrix} = \begin{bmatrix} -1,2644 \\ -1,2644 \\ -1,2644 \\ -1,6550 \\ -0,1824 \end{bmatrix} \qquad X = \begin{bmatrix} \ln\varsigma_d \\ \ln\varsigma_b \\ \ln\varsigma_L \\ \ln\varsigma_V \\ \ln\varsigma_n \end{bmatrix} = \begin{bmatrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0\\ 1&1&0&1&1\\ 0&0&0&1&0 \end{bmatrix}^{-1} \begin{bmatrix} -1,2644 \\ -1,2644 \\ -1,2644 \\ -1,6550 \\ -0,1824 \end{bmatrix}$$

$$X = \begin{bmatrix} -1,2646 \\ -1,2644 \\ -1,2644 \\ -0,1823 \\ 1,0564 \end{bmatrix} \Rightarrow\Rightarrow\Rightarrow \begin{bmatrix} \varsigma_d \\ \varsigma_b \\ \varsigma_L \\ \varsigma_V \\ \varsigma_n \end{bmatrix} = \begin{bmatrix} 0,2824 \\ 0,2824 \\ 0,2824 \\ 0,8333 \\ 2,8761 \end{bmatrix}$$

$$J_D = J_{D\_ref}\,\varsigma_n\varsigma_d\varsigma_b \qquad B_{max} = B_{max\_ref}\,\varsigma_V.(\varsigma_L)^{-1}$$
$$J_D = 1,0320 Amper \qquad B_{max} = 590,2\ Gauss$$

$$\varepsilon_a = \varepsilon_{a\_ref}.(\varsigma_V)^{-1} \qquad \varepsilon_i = \varepsilon_{i\_ref}.(\varsigma_V)^{-1}$$
$$\varepsilon_a = 0,0648 \qquad \varepsilon_i = 0,1632$$

$$\varepsilon_w = \varepsilon_{w\_ref}.(\varsigma_L).(\varsigma_V)^{-1}.(\varsigma_b)^{-1} \qquad \lambda_{diff} = \lambda_{diff\_ref}.(\varsigma_n)^{-1/2}(\varsigma_L)^{-1/2}(\varsigma_V)^{-1/2}$$
$$\varepsilon_w = 0,3 \qquad \lambda_{diff} = 0,2188$$

$$\eta_{losses} = 1 - \varepsilon_w - \varepsilon_a - \varepsilon_i \qquad \lambda_i = \lambda_{i\_ref}.(\varsigma_n)^{-1}(\varsigma_L)^{-1}$$
$$\eta_{losses} = 0,4720 \qquad \lambda_i = 0,7633$$

After determining the discharge current, the power and efficiency of the scaled thruster, it is straightforward to get all the related performance parameters:

Ion mass flowrate:

$$\dot{m}_i = \frac{\eta_J J_D}{e.(1+\alpha)} M_{Xe}$$
$$\eta_J = 0,81$$
$$J_D = 1,0320 Amper$$
$$\alpha = 0,15$$
$$\dot{m}_i = 9,8584 * 10^{-7} kg/\sec$$
$$\dot{m}_i = 0,9858\ mg/\sec$$

total mass flow rate:

$$\dot{m}_{tot} = \frac{\dot{m}_i}{\eta_m}$$
$$\dot{m}_{tot} = 1,01\ mg/\sec$$

$$\alpha = \frac{\dot{m}_i^{++}}{\dot{m}_i}$$

Alpha is the doubly ionized particle fraction.

Ion exhaust velocity

$$v_{ion} = (1-\alpha)\sqrt{\frac{2eV\eta_{losses}\eta_{vel}\eta_\varphi}{M_{Xe}}} + \alpha\sqrt{\frac{2.(2e)V\eta_{losses}\eta_{vel}\eta_\varphi}{M_{Xe}}}$$
$$\eta_{losses} = 0,56$$
$$\eta_{vel} = 0,95$$
$$\eta_\varphi = 0,95$$
$$V = 250\ Volts$$
$$v_{ion} = 1,3310 * 10^4\ m/\sec$$

Total Thrust:

$$T = \dot{m}_i v_{ion}$$
$$\mathrm{T} = 13,1217 * 10^{-3}\ \mathrm{N}$$
$$\mathrm{T} = 13,1217\ mN$$

$$v_{eff} = \eta_m.v_{ion}$$
$$\eta_m = 0,98$$
$$v_{eff} = 1,3044 * 10^4\ m/\sec$$

$$Isp = \frac{v_{eff}}{g_o}$$
$$Isp = 1330 \sec$$

$$\eta_{total} = \eta_{losses}\eta_J\eta_{vel}\eta_\varphi\eta_m$$
$$\eta_{total} = 0,3381$$

Thruster/Parameter	**SPT-30**	**TRK-30 (predicted values from scaling algorithm of Andrenucci)**
d_{ch} (mm)	**30**	30
d(mm)	24	24
b(mm)	6	4,2360
L(mm)	11	6,213
V(Volts)	**250**	**250**
P(Watt)	**258**	**258**
Mass flow rate (mg/sec)	0,98	1,01
T(mN)	13,2	13,122
I_{sp}(sec)	1234	1330
Eta(efficiency)	0,31	0,3381

Table 4: The predicted performance parameters for TRK-30 in the right coloumn

Conclusions and Future Work

The scaling and proper magnetic field topography of the proposed prototype Hall Effect Thruster TRK-30 were discussed. After scaling of TRK-30 with the help of experimentally mature SPT-100 and by using geometric scaling laws, the dimensions of the discharged channel are determined that mean dimeter “d” is 24 mm, channel widht “b” is 4.236 mm and channel lenght “L” is 6.213 mm. By using these dimensions, operating voltage, and power level we have determined the expected thruster performance parameters, such as thrust, specific impulse, mass flow rate and total efficiency. In order to sustain these performance parameters, of course, the magnetic field distribution and the location where the maximum radial magnetic field is should resemble that of SPT-100 has. In that discusion we use the magnetic field data which the P5 5 kW Hall Effect Thruster has due to lack of information of SPT-100 magnetic field. On the other hand we have known the production scientist of P5 have tried to resemble the magnetic field distribution of the P5 like as that of SPT-100. Because of that we can compare the P5 magnetic data and TRK-30. As future work, the production of the TRK-30 will be performed and performance parameters will be measured.

[1] Misuri, T., Battista, F., Barbieri, C., De Marco, E. A., and Andrenucci, M., “High power Hall thruster design options," Proceedings of the 30th International Electric Propulsion Conference, 2007.

[2] Gulczinski III, F. S., Examination of the structure and evolution of ion energy properties of a 5 kW class laboratory Hall Effect thruster at various operational conditions, Ph.D. thesis, The University of Michigan, 1999.

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